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A SIMPLE METHOD OF DETERMINING
THE EFFECT OF REACTOR TEMPERATURE
ON CRITICALITY CONDITIONS

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PREFACE

This study extends the work reported in RAND Memorandums RM-2280, A Correlation of the Critical Conditions for Homogeneous Bare Reactors, and RM-2940-PR, A Method of Correlating Critical Conditions of Homogeneous Bare Reactors Containing a Resonance Absorber, and shows how the effect of reactor temperature on criticality can be computed by simple hand calculation methods while retaining the accuracy of multi-group electronic-computer calculations.

The methods presented in this series of studies are intended to assist engineers in making feasibility studies of reactor systems by providing a ready insight into the effect of important parameters on criticality conditions.

SUMMARY AND CONCLUSIONS

As part of a series of studies designed to simplify reactor anal-
ysis for use in engineering evaluations, an analysis is made of the
effect of reactor temperature on criticality conditions, and simple
relations are obtained for calculating these effects.

The buckling (and hence reactor size for criticality) may be ex-
pressed as a function of the fission, capture, and scatter cross sec-
tions of the reactor materials averaged over the flux spectrum. If
these parameter averages are known for one reactor temperature, then
the simple relations derived in this study allow computation for other
reactor temperatures. A substantial simplification of the analysis
results from the assumption that the major part of the influence of
reactor temperature comes from the changes in the cross sections and
in the values of the flux distribution in the vicinity of thermal en-
ergy. Good agreement is obtained between the values computed from the
equations and the values obtained from an 18-group machine calculation
performed for a series of reactors covering a temperature range from
25°C to 1450°C.

ACKNOWLEDGMENT

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CONTENTS

SYMBOLS

B Buckling, cm^{-1}

E Neutron energy, ev

F_3 $\displaystyle\sum_{i=1}^{m} (\nu\sigma_{fu} - \sigma_{au}) \frac{\Phi_i}{\Phi_T}$

F_4 Ratio of mean effective value of σ_a to its thermal value,

$$\sum_{i=1}^{m} \frac{\sigma_{ami}}{\sigma_{amo}} \frac{\Phi_i}{\Phi_T}$$

f_i Fraction of the neutrons born from fission into the ith group

h Height of cylinder, cm

N Number of atoms or molecules per unit volume

P Flux-spectrum parameter (see Eq. (36) and associated discussion)

R_c Radius of cylinder, cm

T Reactor temperature, ^oC

γ See Eq. (9)

μ_e Ratio of neutrons scattered from epithermal group to thermal group per scattering collision in the epithermal group

μ_f Ratio of neutrons scattered from fast group to epithermal group per scattering collision in the fast group

λ_t Transport mean-free path, cm

ν Number of neutrons born from fission

Σ Macroscopic cross section

Σ_{sme} Macroscopic scatter cross section of moderator for epithermal flux

Σ_{smf} Macroscopic scatter cross section of moderator for fast flux

σ Microscopic cross section

Φ Neutron flux

Φ_e Epithermal flux

Φ_f Fast flux

Φ_o Thermal flux

Φ_T Total flux; $\Phi_o + \Phi_e + \Phi_f$

Ψ $\dfrac{\mu_f \sigma_{smf}}{\mu_e \sigma_{sme}}$

Subscripts

a Absorption (except where otherwise defined)

a Quantities evaluated at reference temperature

f Fission (except where otherwise defined)

i Group i

m Moderator

o Thermal-energy parameters

r Radiative capture

s Scatter

t Transport

T Total

u Uranium 235

x Additional reactor material, $1/\sqrt{E}$ absorber

I. INTRODUCTION

This Memorandum continues the development of a method for reactor-criticality analysis which would be suitable for exploratory engineering evaluations. The effect of reactor moderator materials on criticality was analyzed in Ref. 1, and the effect of introducing a resonance absorber was analyzed in Ref. 2. The effect of reactor temperature on criticality is investigated here.

The method developed permits the use of simple desk-calculation techniques in the determination of the effect of reactor parameters and reactor temperature on criticality, and it provides insight into the magnitude of these effects. It makes use of a flux-distribution parameter which, for a given value, yields the same spectral-flux distribution for reactors of different materials. Simple equations are derived from an approximate analysis. Their accuracy in predicting criticality is checked by a comparison with the results of an 18-group machine calculation of a series of $Li^7H - U^{235}$ reactors at various moderator-to-uranium mole ratios, and at reactor temperatures from $25^\circ C$ to $1450^\circ C$. The data used in establishing these correlations consist largely of results of 18- and 53-group machine analyses made by George Safonov.[3]

II. ANALYSIS

CRITICALITY EQUATION

With the notation of Refs. 1 and 2, the equations for an m-group solution of a reactor may in general be represented as m simultaneous equations of the form

$$\frac{\lambda_{ti}}{3} \nabla^2 \Phi_i + f_i \, \nu \sum_{j=1}^{m} \Sigma_{fj} \, \Phi_j + \sum_{j=1}^{i-1} \Sigma_{si}^j \, \Phi_j - \Sigma_{si}^* \, \Phi_i - \Sigma_{ai} \, \Phi_i = 0 \quad (1)$$

The mth group is the thermal group and discrete parameters in the thermal group will be given the subscript o.

λ_{ti} = the transport mean free path of group i

Φ_i = the neutron flux of group i

f_i = the fraction of the neutrons that are born in group i

(by definition, $\sum_{i=1}^{m} f_i = 1$)

ν = the number of neutrons released per fission capture of a neutron

Σ_{fj} = the macroscopic fission cross section of U^{235} for neutrons in group i

Σ_{si}^j = the macroscopic cross section for the slowing down of neutrons from group j to group i

Σ_{si}^* = the macroscopic cross section for the slowing down of neutrons from group i to a lower group; this quantity has a value of zero for the thermal-energy group

Σ_{ai} = the macroscopic capture cross section for neutrons in group i

When these m equations (Eq. (1)) are added, the summations of the third terms and fourth terms cancel each other because the total

number of neutrons slowed into all energy groups is equal to the
total number of neutrons slowed out of all energy groups.

This summation, with the further assumption that the neutron-
energy spectrum is independent of position in the reactor, leads to
the following equation

$$\frac{\lambda_t}{3} \nabla^2 \Phi_T + \left[\upsilon \sum_{i=1}^{m} \Sigma_{fi} \frac{\Phi_i}{\Phi_T} - \sum_{i=1}^{m} \Sigma_{ai} \frac{\Phi_i}{\Phi_T} \right] \Phi_T = 0 \qquad (2a)$$

where

$$\Phi_T = \sum_{i=1}^{m} \Phi_i \qquad (2b)$$

and

$$\lambda_t = \sum_{i=1}^{m} \lambda_{ti} \frac{\Phi_i}{\Phi_T} \qquad (2c)$$

Equation (2a) may be written in the usual form involving the
buckling, B

$$\nabla^2 \Phi_T + B^2 \Phi_T = 0 \qquad (3)$$

where

$$\frac{\lambda_t B^2}{3} = \upsilon \sum_{i=1}^{m} \Sigma_{fi} \frac{\Phi_i}{\Phi_T} - \sum_{i=1}^{m} \Sigma_{ai} \frac{\Phi_i}{\Phi_T} \qquad (4)$$

For the case under discussion, the reactor is made up of U^{235}, a moderator, and a $1/\sqrt{E}$ absorber, and Eq. (4) may be written

$$\frac{\lambda_t B^2}{3N_u} = \sum_{i=1}^{m} (\nu\sigma_{fui} - \sigma_{aui}) \frac{\Phi_i}{\Phi_T} - \frac{N_m}{N_u} \sum_{i=1}^{m} \sigma_{ami} \frac{\Phi_i}{\Phi_T} - \frac{N_x}{N_u} \sum_{i=1}^{m} \sigma_{axi} \frac{\Phi_i}{\Phi_T} \quad (5)$$

where

σ_{fu} = the microscopic fission cross section of U^{235}

σ_a = the microscopic absorption cross section

$u = U^{235}$

m = moderator

$x = 1/\sqrt{E}$ absorber

EFFECT OF TEMPERATURE ON URANIUM CROSS-SECTION TERM

If the temperature of the reactor is changed, the capture cross sections in the thermal group and the spectral-flux distribution are changed. The effect of reactor temperature is important for highly thermal reactors and tends to decrease as N_m/N_u decreases. At the low values of N_m/N_u where the relative number of captures of neutrons at thermal energy is small, the effect of reactor temperature on the value of the terms in Eq. (5) is negligible. Thus, in obtaining expressions for the effect of reactor temperature, simplifying assumptions that are accurate in the highly thermal end of the N_m/N_u range will be made.

The first term on the right-hand side of Eq. (5) can be written in terms of its value at a reference temperature and of corrections for the effect of the change in temperature. A change in the percentage

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of thermal flux is accompanied by an equal but opposite change in the
remainder of the flux. The change in the remainder of the flux is
spread out over a large range of energy levels so that any one energy
band receives only a small contribution. Although there are resonance-
energy bands in U^{235} that have a substantial fission and capture cross
section, these receive so small a contribution of the flux increment
that they have only a small effect on the summation term under dis-
cussion. Thus, a good approximation can be obtained for the effect
of temperature by considering only the contribution of fissions and
captures in the vicinity of the thermal-energy level.

Consider a change in reactor temperature from a reference tem-
perature, T_a, to a higher temperature, T. In the case where the re-
actor temperature is T_a, the mth group, which is the thermal group,
will be taken as corresponding to the temperature T_a and the $m - 1^{th}$
group will be taken as having an energy corresponding to the higher
temperature, T. For the reactor at temperature T, the thermal group
will correspond to the same energy as that of the $m - 1^{th}$ group for
the reactor at temperature T_a but will still be designated the mth
group to retain the notation that the mth group is the thermal group.

If, in accordance with the previous discussion, for the first
term on the right-hand side of Eq. (5) we consider only changes coming
from groups whose energies correspond to the temperatures T_a and T, we
obtain

$$\sum_{i=1}^{m} (\nu\sigma_{fui} - \sigma_{aui}) \frac{\Phi_i}{\Phi_T} - \sum_{i=1}^{m} (\nu\sigma_{fui} - \sigma_{aui}) \left(\frac{\Phi_i}{\Phi_T}\right)_a = \left[\nu\sigma_{fuo} - \sigma_{auo}\right] \frac{\Phi_o}{\Phi_T}$$

$$- \left[\nu\sigma_{fuo} - \sigma_{auo}\right] \left(\frac{\Phi_o}{\Phi_T}\right)_a - \left[\nu\sigma_{fu(m-1)} - \sigma_{au(m-1)}\right]_a \left(\frac{\Phi_{m-1}}{\Phi_T}\right)_a \tag{6}$$

where the subscript a designates parameters associated with the reference reactor temperature. Now we assume that

$$\left(\frac{\Phi_{m-1}}{\Phi_T}\right)_a \cong A \left(\frac{\Phi_{epi}}{\Phi_T}\right)_a \cong AB \left(1 - \frac{\Phi_o}{\Phi_T}\right)_a \tag{7}$$

where A and B are constants. The nearly linear relation between Φ_{epi}/Φ_T and Φ_o/Φ_T will be illustrated later. Furthermore

$$\left[\nu\sigma_{fu(m-1)} - \sigma_{au(m-1)}\right]_a = \sqrt{\frac{E_o}{E_{m-1}}} \left[\nu\sigma_{fuo} - \sigma_{auo}\right]_a \tag{8}$$

Substituting from Eqs. (7) and (8) into Eq. (6), we obtain

$$F_3(P) - F_3(P)_a = \left[\nu\sigma_{fuo} - \sigma_{auo}\right] \frac{\Phi_o}{\Phi_T} - \left[\nu\sigma_{fuo} - \sigma_{auo}\right]_a$$

$$\left[\left(\frac{\Phi_o}{\Phi_T}\right)_a (1 - \gamma) + \gamma\right] \tag{9}$$

where

$$F_3(P) \equiv \sum_{i=1}^{m} (\nu\sigma_{fui} - \sigma_{aui}) \frac{\Phi_i}{\Phi_T}$$

and

$$\gamma = AB \sqrt{\frac{E_o}{\overline{E}_{m-1}}}$$

the quantity γ may be conveniently plotted as a function of $\sigma_{fuo}/(\sigma_{fuo})_a$.

In the determination of γ the energy interval between temperature T_a and

T can, if it is large, be broken into more than two groups. A plot of

γ against $\sigma_{fuo}/(\sigma_{fuo})_a$ obtained from the 18-group analysis of series of a

LiH - U^{235} reactors will be shown later, and the accuracy of Eq. (9) in

computing the effect of reactor-temperature change on the value of

$$\sum_{i=1}^{m} (\nu\sigma_{fui} - \sigma_{aui}) \frac{\phi_i}{\overline{\phi}_T}$$

will be illustrated.

EFFECT OF TEMPERATURE ON MODERATOR CROSS-SECTION TERM

The effect of temperature on the terms in Eq. (5) for the $1/\sqrt{E}$

absorbers will be illustrated for the case of the term

$$\sum_{i=1}^{m} \sigma_{ami} \frac{\phi_i}{\overline{\phi}_T}$$

For a $1/\sqrt{E}$ absorber this term may be written as

$$\sum_{i=1}^{m} \sigma_{ami} \frac{\phi_i}{\overline{\phi}_T} = \sigma_{amo} \sum_{i=1}^{m} \frac{E_o}{E_i} \frac{\phi_i}{\overline{\phi}_T} \qquad (10)$$

If again we consider only groups in the thermal-energy range and follow

the same procedure applied in the case of the previous term in Eq. (5),

we may write, analogous to Eq. (9),

$$\sigma_{amo} \sum_{i=1}^{m} \sqrt{\frac{E_o}{E_i} \frac{\Phi_i}{\Phi_T}} - \sigma_{amo} \sum_{i=1}^{m} \sqrt{\frac{E_o}{E_i}} \left(\frac{\Phi_i}{\Phi_T}\right)_a =$$

$$\sigma_{amo} \frac{\Phi_o}{\Phi_T} - (\sigma_{amo})_a \left[\left(\frac{\Phi_o}{\Phi_T}\right)_a (1-\gamma) + \gamma\right] \qquad (11)$$

Using the definition of F_4 given in Refs. 1 and 2, namely

$$F_4 = \sum_{i=1}^{m} \sqrt{\frac{E_o}{E_i} \frac{\Phi_i}{\Phi_T}}$$

we may write Eq. (11) as

$$\sigma_{amo} F_4 - (\sigma_{amo} F_4)_a = \sigma_{amo} \frac{\Phi_o}{\Phi_T} - (\sigma_{amo})_a \left[\left(\frac{\Phi_o}{\Phi_T}\right)_a (1-\gamma) + \gamma\right] \qquad (12)$$

The value of γ obtained for the previous case as a function of $\sigma_{fuo}/(\sigma_{fuo})_a$ applies also in the case of Eq. (12) if $\sigma_{fuo}/(\sigma_{fuo})_a$ is replaced by $\sigma_{amo}/(\sigma_{amo})_a$.

An equation analogous to Eq. (12) can also be written for the last term of Eq. (5).

In order to apply Eqs. (9) and (12) it is necessary to determine Φ_o/Φ_T.

EFFECT OF TEMPERATURE ON FLUX DISTRIBUTION

An expression for the value of Φ_o/Φ_T relative to $(\Phi_o/\Phi_T)_a$ which is sufficiently accurate for the present purpose can be obtained by a 3-group analysis. Again let it be assumed that the major difference

occurs in highly thermal reactors. The leakage and capture of neutrons in the fast and epithermal ranges will be neglected. The neutron balance equations for the three energy groups may be written as

$$\nu \, \Sigma_{fuo} \, \Phi_o - \mu_f \, \Sigma_{smf} \, \Phi_f = 0 \tag{13}$$

$$\mu_f \, \Sigma_{smf} \, \Phi_f - \mu_e \, \Sigma_{sme} \, \Phi_e = 0 \tag{14}$$

$$\frac{-B^2 \lambda_t \, \Phi_o}{3} + \mu_e \, \Sigma_{sme} \, \Phi_e - \Sigma_{auo} \, \Phi_o - \Sigma_{amo} \, \Phi_o = 0 \tag{15}$$

where Φ_f is the fast flux, Φ_e is the epithermal flux, and μ_f and μ_e are the removal fractions for the fast and epithermal group, respectively. The fast flux is assumed to cover energies from 10^5 to 10^7 ev, which represents the range of energies for most neutrons at birth from fission. The epithermal flux includes the remainder of the energy spectrum between the fast and thermal bands. From Eq. (14), we find

$$\frac{\Phi_e}{\Phi_f} = \frac{\mu_f \, \Sigma_{smf}}{\mu_e \, \Sigma_{sme}} \tag{16}$$

Equation (13) yields

$$\frac{\Phi_f}{\Phi_o} = \frac{\nu \, \Sigma_{fuo}}{\mu_f \, \Sigma_{smf}} \tag{17}$$

From Eqs. (16) and (17), we have

$$\frac{\Phi_e}{\Phi_o} = \frac{\nu \, \Sigma_{fuo}}{\mu_e \, \Sigma_{sme}} \tag{18}$$

By definition

$$\Phi_T = \Phi_o + \Phi_e + \Phi_f \tag{19}$$

or

$$\frac{\Phi_T}{\Phi_o} = 1 + \frac{\Phi_e}{\Phi_o} + \frac{\Phi_f}{\Phi_o} \tag{20}$$

Substituting from Eqs. (17) and (18), we find

$$\frac{\Phi_T}{\Phi_o} = 1 + \nu \, \Sigma_{fuo} \left[\frac{1}{\mu_e \, \Sigma_{sme}} + \frac{1}{\mu_f \, \Sigma_{smf}} \right] \tag{21}$$

For the reactor at the reference temperature

$$\left(\frac{\Phi_T}{\Phi_o} \right)_a = 1 + \nu \, (\Sigma_{fuo})_a \left[\frac{1}{\mu_e \, \Sigma_{sme}} + \frac{1}{\mu_f \, \Sigma_{smf}} \right] \tag{21a}$$

From Eqs. (21) and (21a), we obtain

$$\frac{\frac{\Phi_T}{\Phi_o} - 1}{\left(\frac{\Phi_T}{\Phi_o} \right)_a - 1} = \frac{\sigma_{fuo}}{(\sigma_{fuo})_a} \tag{22}$$

A plotting of the left-hand side of Eq. (22), computed from machine multigroup analyses, against $\sigma_{fuo}/(\sigma_{fuo})_a$ should give a straight line which has a slope of unity.

Thus, if the results of a reactor analysis at a reference temperature T_a are known, then the criticality conditions at another temperature can be computed by making use of Eqs. (22), (12), (9), and (5).

EFFECT OF TEMPERATURE ON λ_t

The value of λ_t in the thermal-reactor range changes only slowly with changes in flux distribution. Thus, a change in reactor temperature should have only a minor effect on λ_t.

For most moderator materials the effect of reactor temperature on mean free path is negligible. For hydrogenous moderators there is a sufficient change in the transport cross section of hydrogen in the thermal-energy range to cause a small change in the transport mean free path. This correction, obtained by considering only first-order effects, is given by the following approximate relation:

$$\lambda_t - (\lambda_t)_a = \lambda_{to} \frac{\Phi_o}{\Phi_T} - (\lambda_{to})_a \left(\frac{\Phi_o}{\Phi_T} \right)_a + \lambda_{t\;ave} \left[\left(\frac{\Phi_o}{\Phi_T} \right)_a - \frac{\Phi_o}{\Phi_T} \right] \qquad (23)$$

where $\lambda_{t\;ave}$ is the average transport mean free path for energy groups above thermal.

If we assume that the transport mean free path for highly thermal reactors is inversely proportional to the transport cross section for the moderator σ_{tm}, then Eq. (23) may be written

$$\lambda_t - (\lambda_t)_a = (\lambda_{to})_a \left[\left(\frac{(\sigma_{tmo})_a}{\sigma_{tmo}} - \frac{(\sigma_{tmo})_a}{\sigma_{tm\;ave}} \right) \frac{\Phi_o}{\Phi_T} - \left(1 - \frac{(\sigma_{tmo})_a}{\sigma_{tm\;ave}} \right) \left(\frac{\Phi_o}{\Phi_T} \right)_a \right] (24)$$

where

$$(\lambda_{to})_a = \frac{1}{N_m (\sigma_{tmo})_a} \left[1 + \frac{N_u \, \sigma_{tuo}}{N_m (\sigma_{tmo})_a} \right]^{-1}$$

and where $\sigma_{tm\ ave}$ is the average value of a transport crosss section for energies above thermal.

For the $Li^7H - U^{235}$ reactors under consideration, the following values for the transport cross section of the moderator were used in Eq. (24) to compute the values of λ_t at various reactor temperatures from the values at a reference temperature of $25^{\circ}C$.

Transport Cross Section of Moderator (barns)	Temperature ($^{\circ}C$)
$(\sigma_{tmo})_a$ = 28.43	25
σ_{tmo} = 9.91	400
σ_{tmo} = 9.02	1450
$\sigma_{tm\ ave}$ = 4.90	....

As an approximation in the present example, the following value was used:

$$(\lambda_{to})_a = \frac{16.2}{(\sigma_{tmo})_a} = 0.57 \ cm$$

where 16.2 corresponds to a Li^7H density of 0.82 g/cc. This is based on the assumption that the contribution of the U^{235} is small.

A comparison of the values of λ_t computed from Eq. (24) with the values taken from the 18-group analysis will be made later.

Equation (24) does not include the effect on λ_t of the variation of density which is caused by change in temperature. This effect can be introduced by assuming that λ_t varies inversely with density.

From Eqs. (16) and (21) we obtain the following linear relations between the flux components:

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$$\frac{\Phi_f}{\Phi_T} = (1 + \psi)^{-1} (1 - \frac{\Phi_o}{\Phi_T}) \tag{25}$$

$$\frac{\Phi_e}{\Phi_T} = \psi (1 + \psi)^{-1} (1 - \frac{\Phi_o}{\Phi_T}) \tag{26}$$

where

$$\psi = \frac{\mu_f \sigma_{smf}}{\mu_e \sigma_{sme}} \tag{27}$$

REACTOR TEMPERATURE

In order to provide a consistent method of relating the properties of reactors in the thermal group with reactor temperature, the following procedure was somewhat arbitrarily adopted. It was assumed that after a neutron was slowed into the thermal-energy group it would thereafter be considered a member of this group, regardless of its subsequent energy history. Collisions with reactor materials would then impose on the group of so-called thermal neutrons the Maxwellian energy distribution which would include, of course, the high-energy range. The values of the reactor parameters would therefore be evaluated on the basis of this Maxwellian distribution.

The number of collisions of type j per cm^3 per sec experienced by the neutrons in the thermal group on the basis of the foregoing discussion is

$$\int_o^\infty \Sigma_j \Phi \, dE \tag{28}$$

where Φ is the thermal flux per unit energy at energy E, and Σ_j is the macroscopic cross section for the collisions of type j at energy E. The mean value of the parameter Σ_j in the thermal energy group is

$$\overline{\Sigma}_j = \frac{\int_o^\infty \Sigma_j \, \Phi \, d E}{\int_o^\infty \Phi \, d E} \tag{29}$$

The thermal flux per unit of energy on the assumption of a Maxwellian distribution is given by

$$\Phi = \frac{E}{(KT)^2} \, e^{-E/KT} \tag{30}$$

For the case in which Σ_j represents a $1/\sqrt{E}$ absorber, let Σ_a be the macroscopic cross section at an energy equal to KT, where T is the temperature of the reactor.

Then in Eq. (29)

$$\Sigma_j = \frac{\Sigma_a \sqrt{KT}}{\sqrt{E}} \tag{31}$$

Thus, Eq. (29) may be written for the case in which $\Sigma_j = \Sigma_a$ is the absorption cross section

$$\overline{\Sigma}_a = \frac{\int_o^\infty \frac{\Sigma_a \sqrt{KT}}{\sqrt{E}} \frac{E}{(KT)^2} e^{-E/KT} \, d E}{\int_o^\infty \frac{E}{(KT)^2} e^{-E/KT} \, d E} \tag{32}$$

Integration leads to the result

$$\Sigma_a = \frac{\sqrt{\pi}}{2} \Sigma_a \qquad (33)$$

Thus, in the present Memorandum, the mean absorption cross section for the thermal group is taken to be equal to the absorption cross section at energy KT, multiplied by $\sqrt{\pi}/2$.

The thermal-energy group can be looked upon as a well into which the neutrons drop at the end of their slowing-down process. Once in this well, the neutrons are assigned energies in accordance with the Maxwellian distributions. In this respect the thermal-energy group is treated differently from the higher-energy groups. In higher-energy groups, the neutrons are assigned energies within the energy bounds of the group, and the average group parameter is defined by

$$\overline{\Sigma}_{ji} = \frac{\int_{E_{i-1}}^{E_i} \Sigma_{ji} \, \Phi_i \, dE}{\int_{E_{i-1}}^{E_i} \Phi_i \, dE} \qquad (34)$$

where Σ_{ji} and Φ_i are functions of E and the integration is taken over the group limit E_{i-1} to E_i.

The values of the thermal-fission cross sections for U^{235} obtained by means of Eq. (33) are

Reactor Temperature (^{O}C)	σ_{fuo} (barns)	$\left(\dfrac{\sigma_{fuo}}{(\sigma_{fuo})_a}\right)$
25	549	1.000
400	344	0.626
1450	200	0.363

where $(\sigma_{fuo})_a$ is the value of σ_{fuo} at $T = 25^O$C.

III. DISCUSSION OF RESULTS

The results of the foregoing analysis will be compared with the results obtained from an 18-group machine calculation relating to the effect of temperature on the critical conditions and associated parameters of a series of LiH - U^{235} reactors.

The effect of temperature on the thermal-flux ratio Φ_o/Φ_T is given by Eq. (22). The straight line in Fig. 1 is a plotting of Eq. (22). The points represent values taken from the 18-group calculation. On the whole, agreement with the equation is good. Some dispersion occurs at low values of N_m/N_u (e.g., 100) where the assumptions of this analysis are inaccurate. However, in this range the effect of temperature on the parameters of Eq. (5) becomes negligible.

Equation (22) is plotted for more convenient application in Fig. 2, where Φ_o/Φ_T is shown as a function of its value at the reference temperature and of $\sigma_{fuo}/(\sigma_{fuo})_a$.

Figure 3 shows plottings of Φ_f/Φ_T and Φ_e/Φ_T against Φ_o/Φ_T taken from the 18-group calculations; in the region where Φ_o/Φ_T is large, it displays the linear relation predicted by Eqs. (25) and (26). Also as predicted by Eqs. (25) and (26), reactor-temperature variations have little influence.

The ratio of the slopes of the curves for Φ_e/Φ_T and Φ_f/Φ_T gives a value of 0.82 for ψ (see Eq. (27)).

A comparison of the values of Φ_o/Φ_T, computed from the values of $(\Phi_o/\Phi_T)_a$ by using Eq. (22) (or Fig. 2), with values taken from the 18-group analysis is shown in Fig. 4a. The abscissa in Fig. 4a is the flux-spectrum parameter P, which in the present case is given by
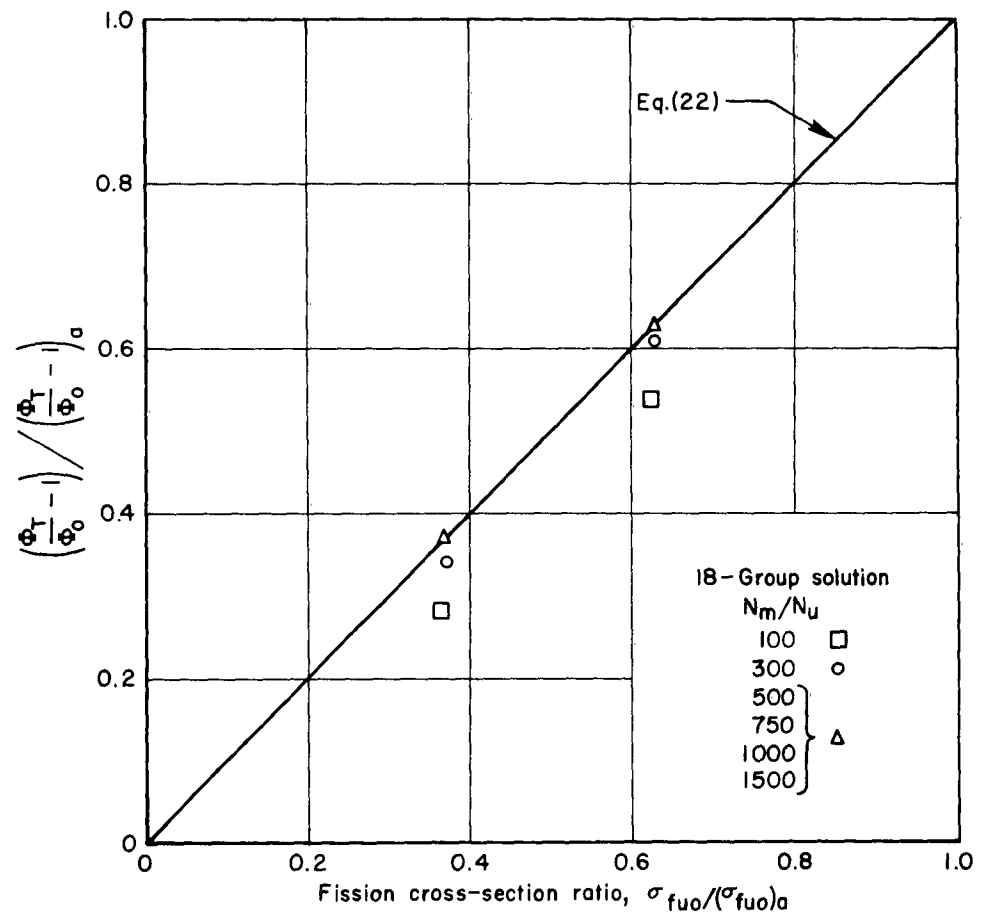
Fig. 1—Correlation of the effect of reactor
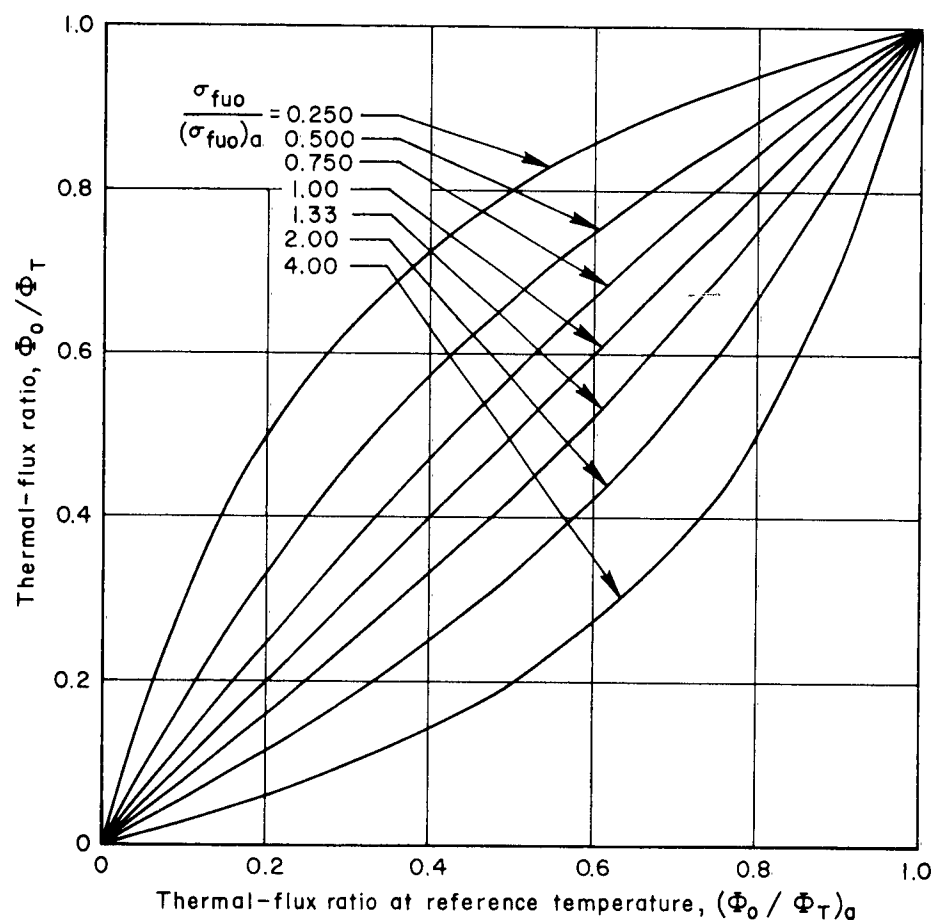temperature on the thermal-flux ratio

Fig. 2 — Graph for obtaining thermal-flux ratio from its value at a reference temperature and the change in temperature (Eq. (22))
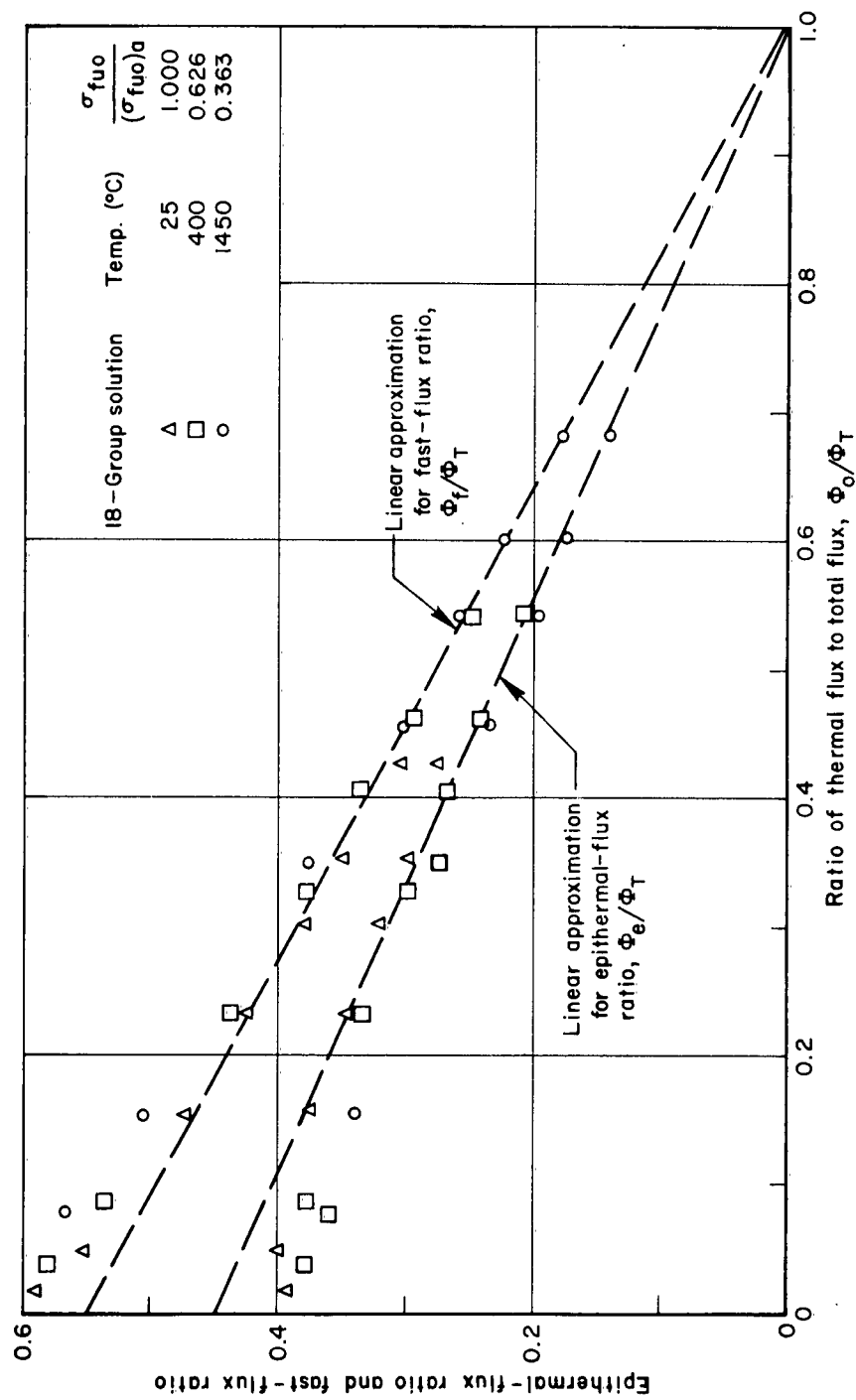
Fig. 3—Variation of epithermal and fast flux with thermal flux
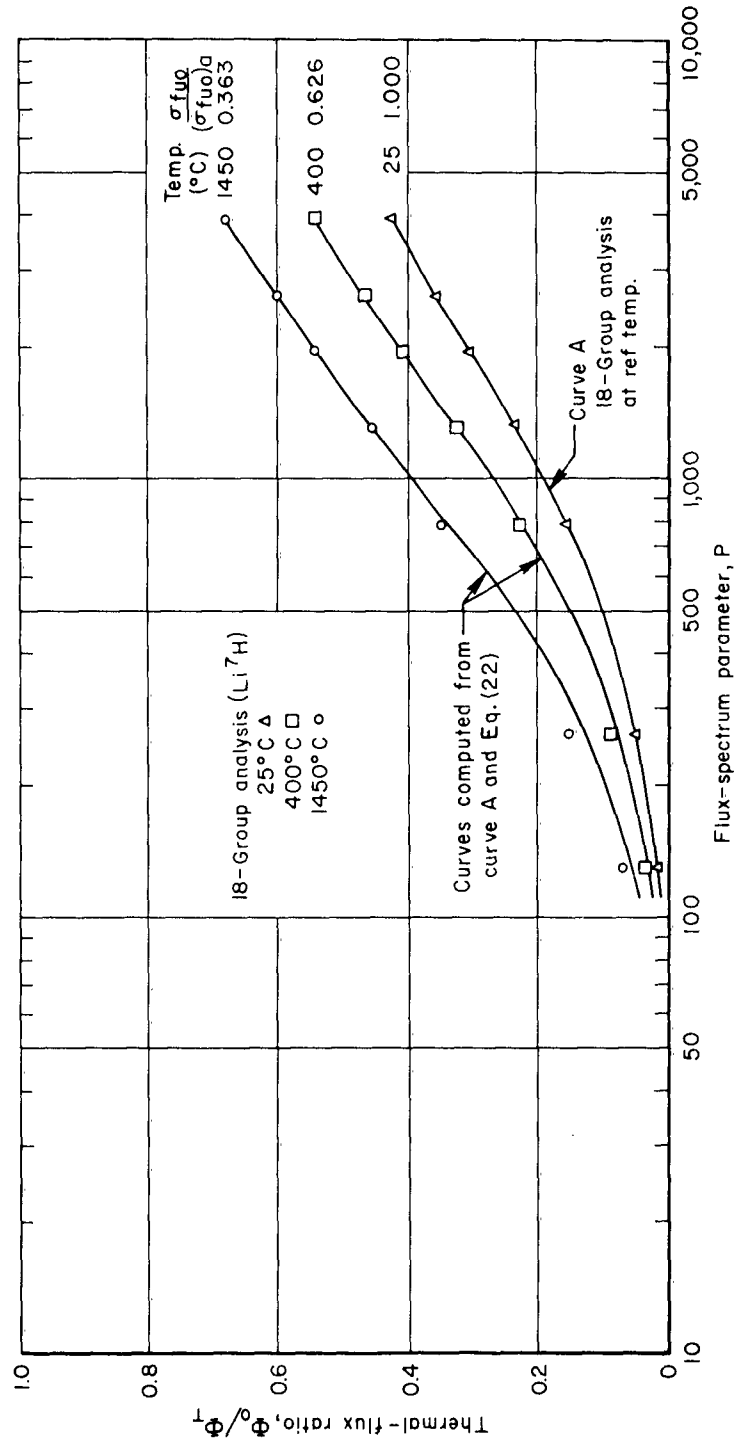
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Fig. 4a—Flux distributions as a function of reactor temperature and flux-spectrum parameter (thermal-flux ratio)

$$P = Y_m N_m / N_u$$

where $Y_m = 2.6$ for a reactor containing LiH and U^{235}. (A more de-
tailed discussion of this parameter is given in Refs. 1 and 2.) The

reference temperature was taken as $25^o C$. Good agreement of the points

with the lines is noted even at the low values of N_m/N_u at which the

analysis is invalid. The values of Φ_e/Φ_T and Φ_f/Φ_T are shown in Figs.

4b and 4c, respectively; these curves also indicate that the effect

of change in reactor temperature tends to disappear at low values of

N_m/N_u.

With the values of Φ_o/Φ_T given by Eq. (22) (or Fig. 2) we can

now, by means of Eq. (9), compute the value of the U^{235} term in Eq.

(5) from its value at the reference reactor temperature. The curves

in Fig. 5 for temperatures $400^o C$ and $1450^o C$ were computed in this

manner for the $Li^7 H - U^{235}$ reactors. The value of γ was taken from

Fig. 6. The points represent values taken from the 18-group solution.

Again the effect of reactor temperature tends to disappear at low

values of P, where the analysis for temperature change becomes in-

valid. Good agreement between the results of the 18-group machine

calculations and the present analysis is obtained over the entire

range of values of P.

The values of the moderator term in Eq. (5) for temperatures of

$400^o C$ and $1450^o C$ computed by means of Eq. (12) and Eq. (22) (or Fig. 2)

are shown in Fig. 7. Again good agreement between these values and the

values taken from the 18-group solution is noted over the entire range

of values of P. The values of F_4 can be used for other $1/\sqrt{E}$ materials

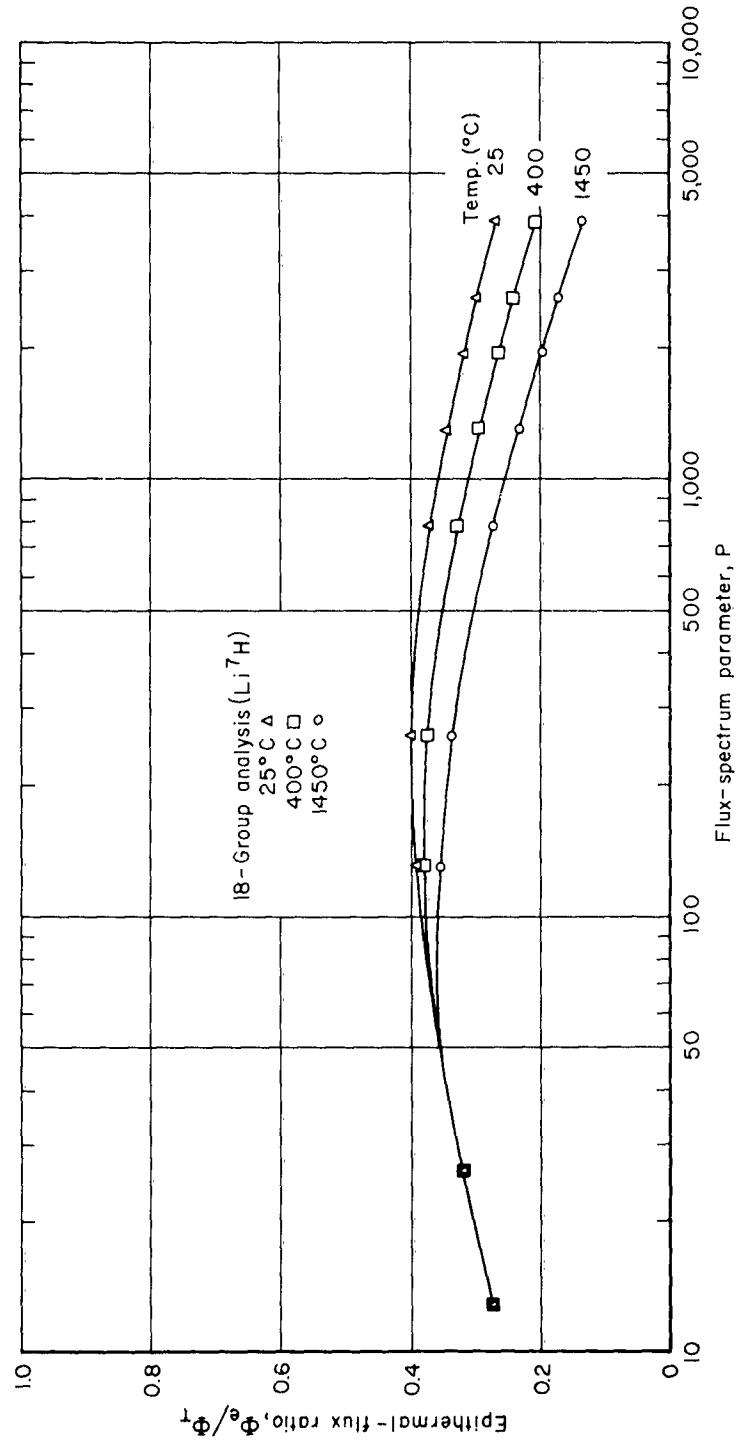in the reactor, such as material x in Eq. (5).

Fig. 4b— Flux distributions as a function of reactor temperature and flux-spectrum parameter (epithermal-flux ratio)
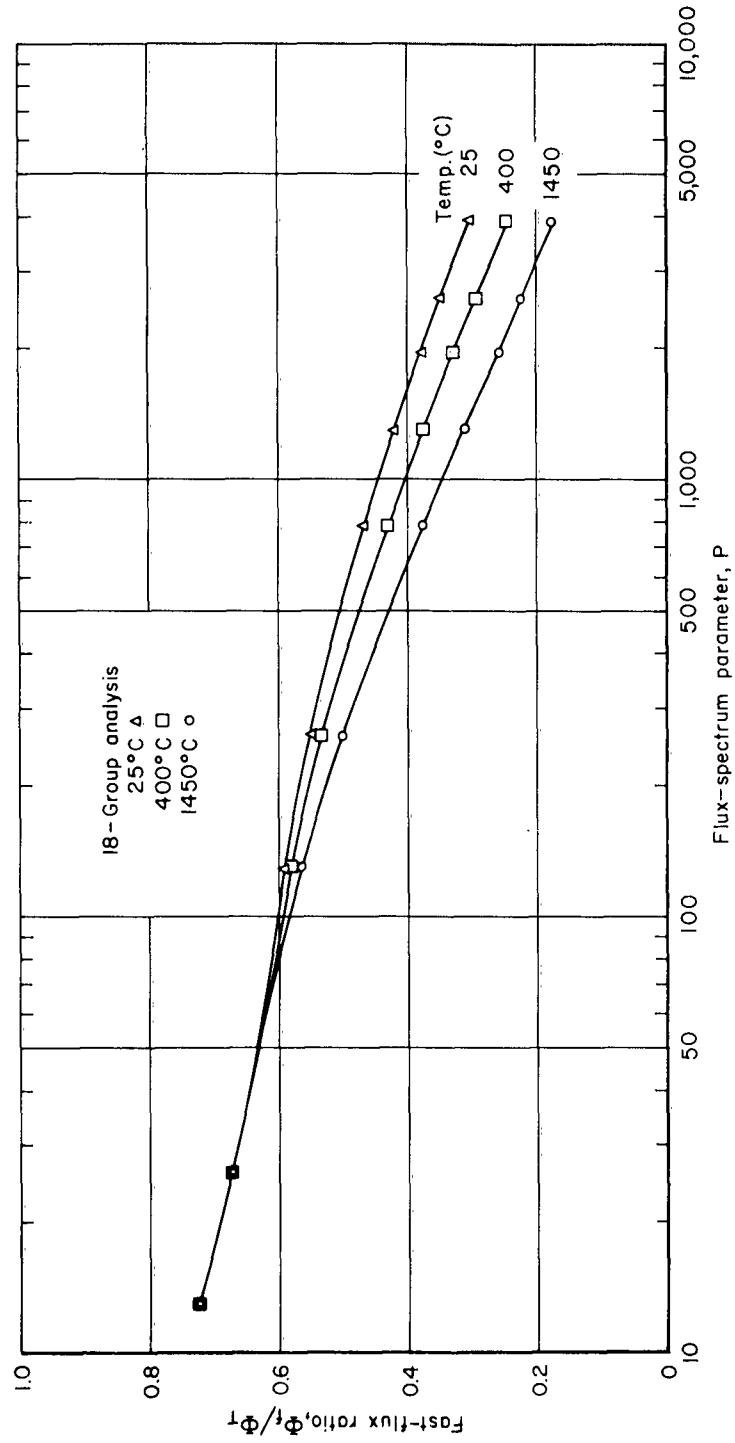
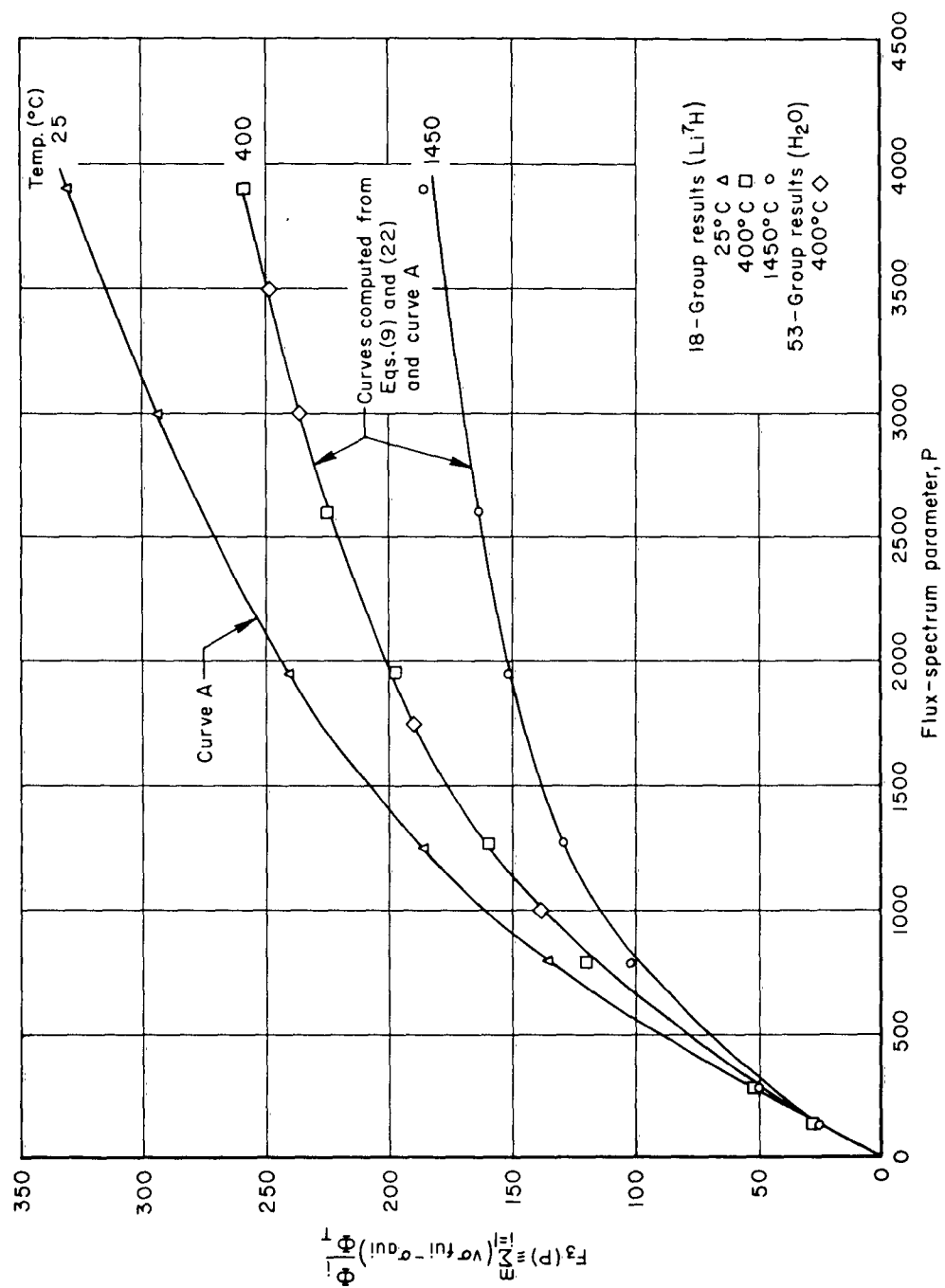Fig. 4c—Flux distributions as a function of reactor temperature and flux-spectrum parameter (fast-flux ratio)

Fig. 5—Effect of reactor temperature on net neutron-generation term
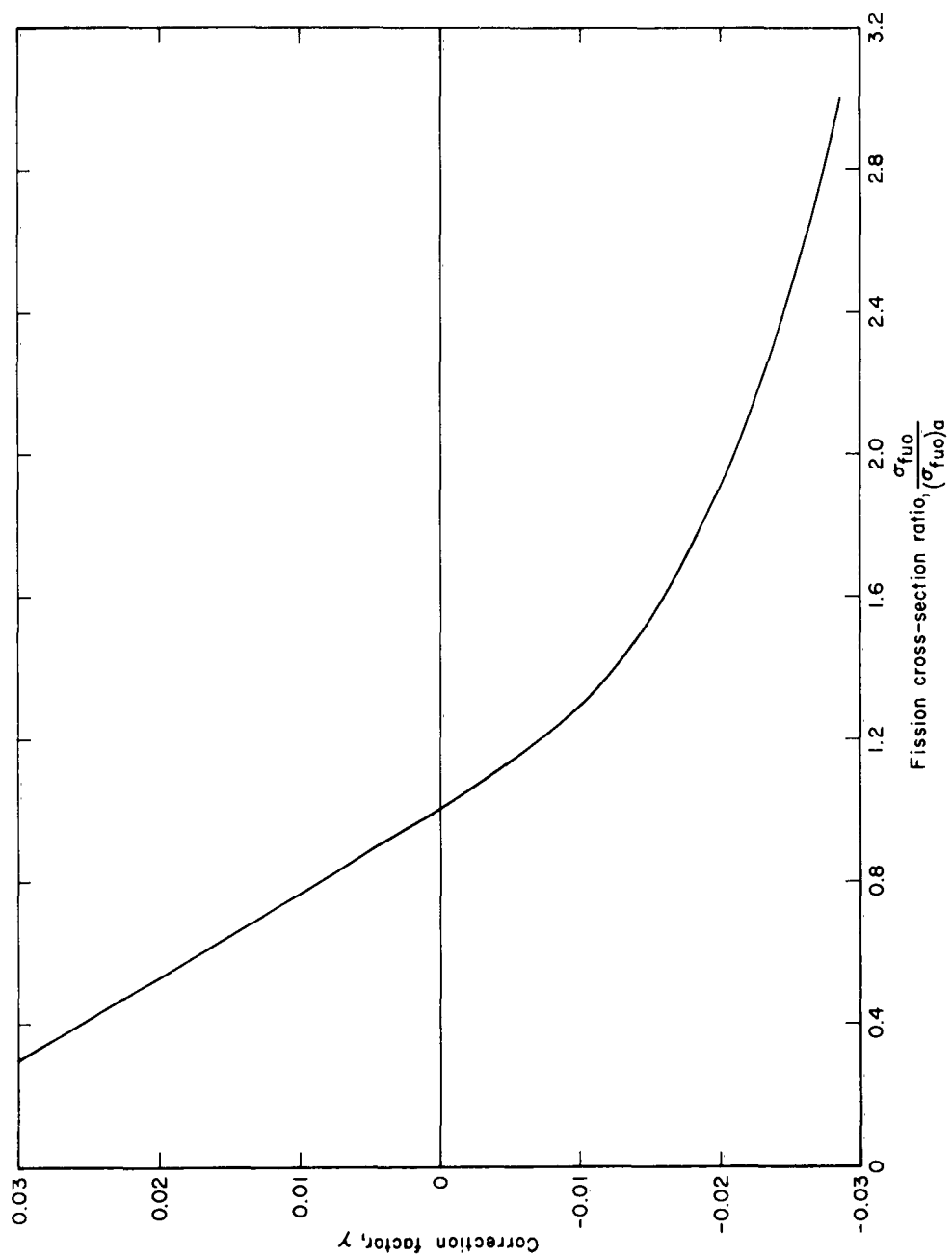
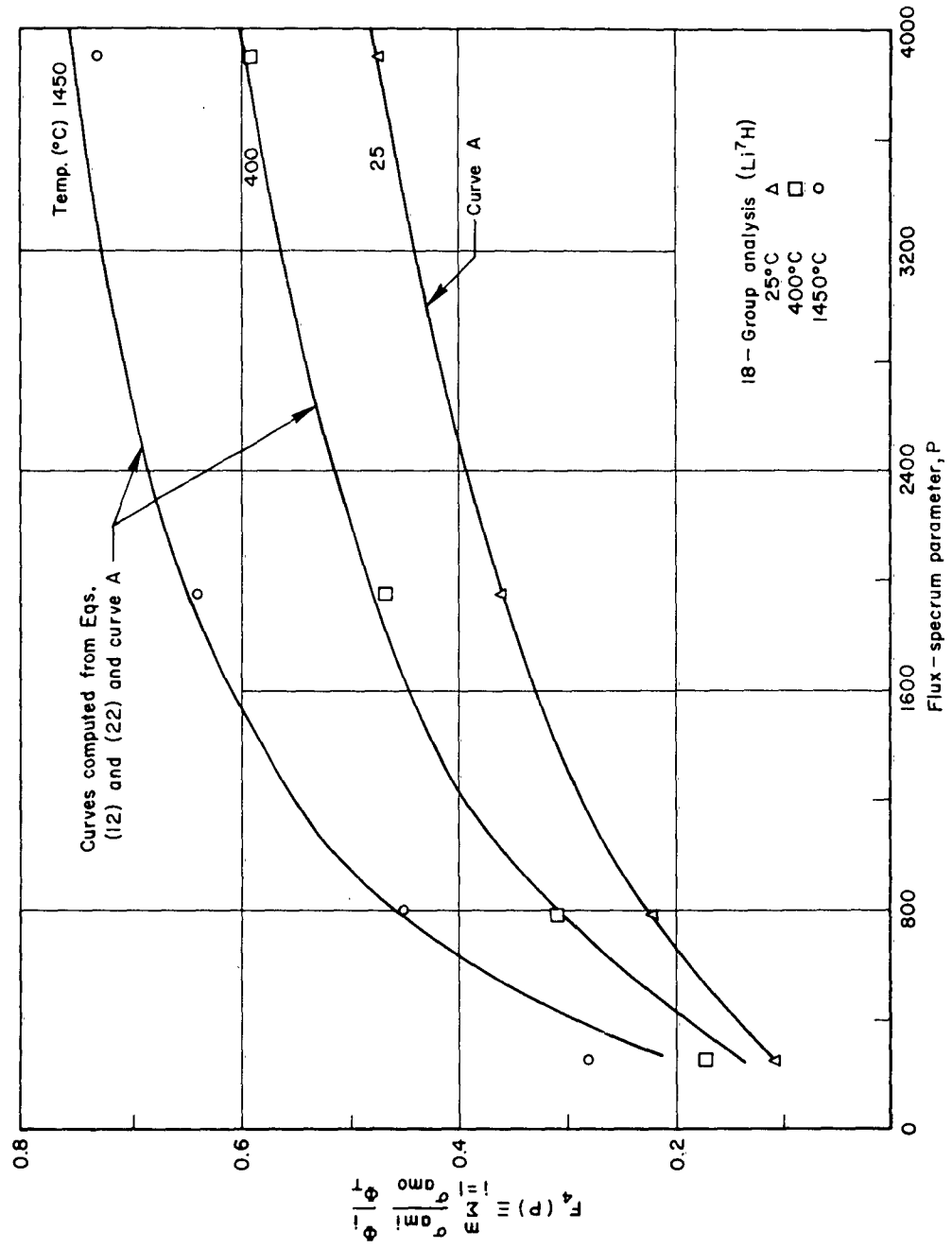Fig. 6—Correction factor γ for Eqs. (9) and (12)

Fig. 7—Effect of reactor temperature on moderator-absorption term

Also shown in Fig. 5 are values taken from a 53-group analysis of an H_2O - U^{235} reactor. In this case the value of Y_m is 5.2. The equations derived in this Memorandum can be used as illustrated in order to obtain the temperature corrections for reactors which contain other moderator materials than those discussed here. Furthermore, once the effect of temperature on the terms of Eq. (5) has been established, their values corresponding to the proper value of P can be read from Figs. 5 and 7 for other reactor materials. This is discussed in Refs. 1 and 3.

The values of λ_t used in Eq. (5) are shown in Fig. 8 both as computed by Eqs. (22) and (24) from the values of λ_t at the reference temperature and as taken from the 18-group solution. The effect of change in density on λ_t is not included in the values given in Fig. 8.

The application of the present method to the calculation of reactor criticality can now be summarized. Equation (5) may be written as

$$\frac{\lambda_t B^2}{3N_u} = F_3(P) - \frac{N_m}{N_u} \sigma_{amo} F_4(P) - \frac{N_x}{N_u} \sigma_{axo} F_4(P) \tag{35}$$

The flux-spectrum parameter, P, can be computed from

$$P = Y_m N_m / N_u \tag{36}$$

for reactors containing only moderator and U^{235}. Reactors having the same value of P have nearly the same flux spectrum, regardless of composition. (This parameter is discussed in greater detail in Refs. 1 and 2.) The values of Y_m are
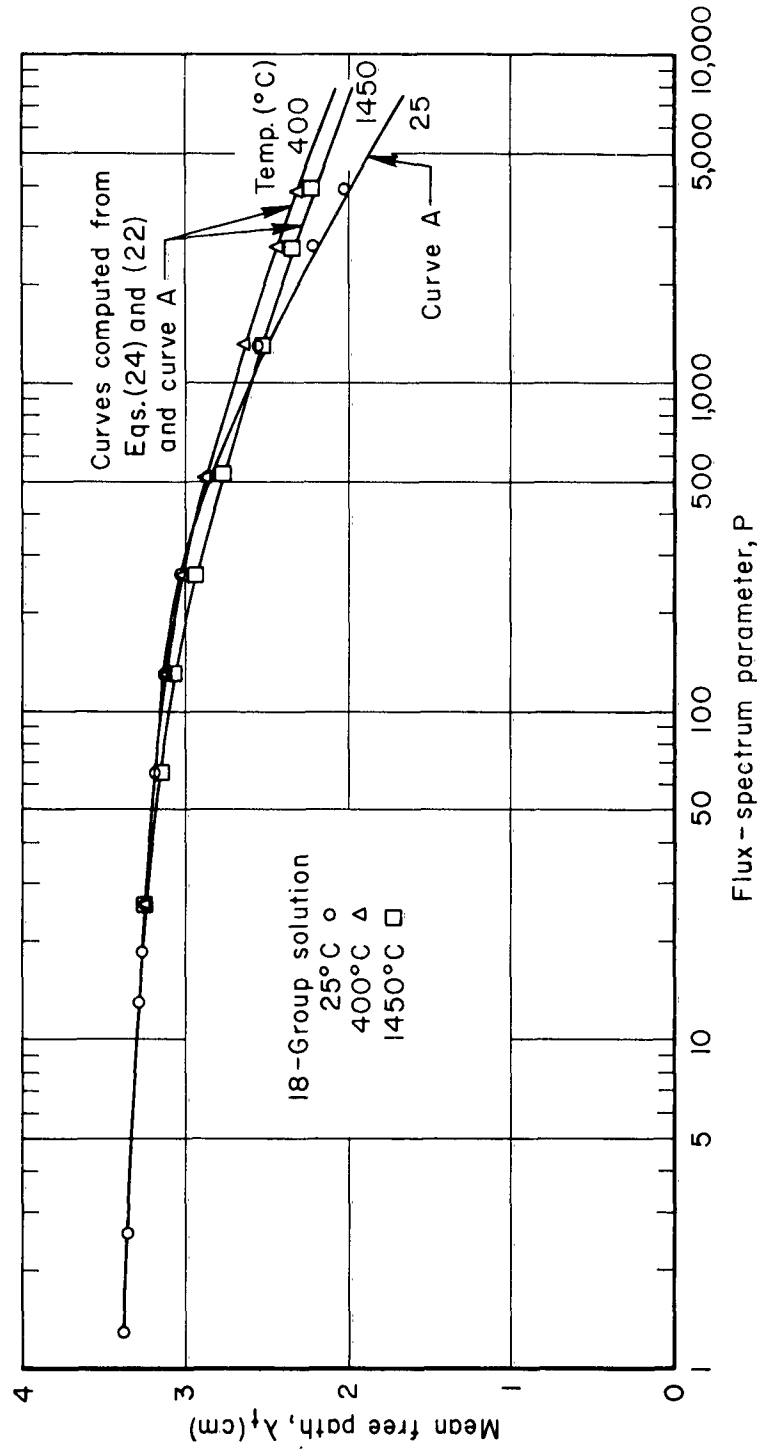
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Fig. 8—Effect of reactor temperature on transport mean free path, λ_t
(material density constant at value corresponding to 25°C)

Moderator	Y_m
D_2O	1.0
H_2O	5.2
Be	0.24
BeO	0.32
C	0.14
Li^7H	2.6

The computation of P for reactors that include a resonance absorber is described in Ref. 3. The value of $F_3(P)$ corresponding to the appropriate values of P and reactor temperature can now be read in Fig. 5. For reactors having a value of P which is less than 200, the value of $F_3(P)$ is independent of temperature and can be obtained from the curves of Refs. 2 and 3, where logarithmic scales are used for more accurate presentation of the low P-range. The quantity $F_4(P)$ corresponding to the appropriate values of P and reactor temperature can be read from Fig. 7 and can be introduced into Eq. (35). The values of σ_{amo} and σ_{axo} are the thermal-absorption cross sections at the reactor temperature. The value of λ_t can be obtained from Fig. 8 for Li^7H and from Refs. 2 and 3 for other moderators. The effect of temperature on λ_t for these other moderators can be computed from Eq. (24). Thus for any reactor composition, i.e., for specified values of N_u, N_m/N_u, and N_x/N_u, the value of the buckling, B, can now be computed from Eq. (35). The effect on B of the change in material density caused by temperature is introduced by using the appropriate values of N_u, N_m, and N_x. The value of λ_t in Fig. 8 corresponds to the material densities at the

reference temperature; as the relation λ_t is inversely proportional to density, this value can therefore be corrected to the density at the desired temperature. The critical size of the reactor can be determined from the value of B by means of simple relations for the standard reactor shapes. For example

Spherical reactor

$$\text{Critical Radius} = \frac{\pi}{B} - \frac{2}{3}\lambda_t$$

Cylindrical reactor

$$\left(\frac{0.7655}{R_c + \frac{2}{3}\lambda_t}\right)^2 + \frac{1}{\left(h_c + \frac{4}{3}\lambda_t\right)^2} = \frac{B^2}{\pi^2}$$

where R_c is the critical radius and h_c is the critical height.

The values in this study and those in Refs. 1 and 2 relate to reactors containing U^{235}. The corresponding values for reactors based on other fissile materials can be computed by means of the equations in these studies.

REFERENCES

1. Pinkel, B., and G.B.W. Young, <u>A Correlation of the Critical Conditions for Homogeneous Bare Reactors</u>, The RAND Corporation, RM-2280 (ASTIA No. AD 225892), October 29, 1958.

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3. Safonov, George, <u>Critical Mixtures of Uranium and 500°F Light Water</u>, The RAND Corporation, R-279, January 8, 1955.